Geometric Satake.

* Our goal this seminar is to prove the following theorem. Let $G/\mathbb{C}$ be a reductive group. Then there is an equiv.

$$\mathrm{Rep}_{G^\vee} \cong \mathrm{Perv}_{L^+G}(\mathrm{Gr}_G) \qquad (*)$$

between REPRESENTATIONS OF THE LANGLANDS DUAL GROUP ($G^\vee$) and PERVERSE SHEAVES on the AFFINE GRASSMANNIAN OF G.

* THE Equivalence above can be understood as a baby case of geometric langlands. Most importantly, it constructs a bridge between the „algebraic" LHS and the „geometric" RHS. It is a remarkable theorem and one of the few ways we know how to understand (=characterize) the langlands dual gp.

* Our initial goal will be to understand the main players in the equation $(*)$.

We will need some time unravelling ~~it~~ esp. what kind of object Gr_G is. it is an „infinite dimensional" scheme. Intuitively, it is the „loop space" of G ($\cong_{top} \Omega G$) and hence we may think of it as a function space.

to fully understand Gr_G, we write it as a colim of finite dimension varieties (= SCHUBERT VARIETIES.) these are usually singular, hence we need to replace the notion of local system in it with PERVERSE SHEAVES, another central notion for us. For this we need to study a bit of t-structures on Δ cats and the gluing theorem of such.

Finally, the theorem is proven as a consequence of the TANNAKIAN formalism. Namely we recognize in $Perv_{L^+G}(Gr_G)$ the formal ~~cat~~ categorical properties making it into Rep_H for some algebraic! group H. then we simply need to understand the root DATA associated to this G to finish.