Periods and Nori motives

Periods:

1) A period is a complex number defined as an integral $\int_\gamma \omega$ where ω is some algebraic differential form over some algebraic variety (defined over $\mathbb{Q}$) and γ is a certain integration domain.

2) (Pure) Hodge structure: $(H_B, H_{dR}, comp_{B,dR})$ with
   - ·) H_B finite dim. $\mathbb{Q}$-vector space;
   - ·) H_{dR} finite dim. $\mathbb{Q}$-vector space with an exhaustive decreasing filtration $F^\bullet$;
   - ·) $comp_{B,dR}: H_{dR} \otimes_{\mathbb{Q}} \mathbb{C} \to H_B \otimes_{\mathbb{Q}} \mathbb{C}$ isomorphism of $\mathbb{C}$-vector spaces

   such that there is $n \in \mathbb{Z}$ for which the induced filtration on $H_B \otimes_{\mathbb{Q}} \mathbb{C}$ satisfies $\forall p$
   $H_B \otimes_{\mathbb{Q}} \mathbb{C} = F^p(H_B \otimes_{\mathbb{Q}} \mathbb{C}) \oplus \overline{F^{n-p+1}(H_B \otimes_{\mathbb{Q}} \mathbb{C})}$.

   Mixed Hodge structures: iterated extensions of Hodge structures.

   Example: X smooth affine variety over $\mathbb{Q}$. Then there is a mixed Hodge structure
   $(H^p_{sing}(X(\mathbb{C}), \mathbb{Q}),\ H^p_{dR}(X, \mathbb{Q}),\ comp_{B,dR})$
   where $comp_{B,dR}$ is induced by the pairing $H^p_{dR}(X,\mathbb{Q}) \otimes \mathbb{C} \times H^{sing}_p(X(\mathbb{C}), \mathbb{Q}) \otimes \mathbb{C} \to \mathbb{C}$

   $$(\omega, \gamma) \longmapsto \int_\gamma \omega$$

   In general, $comp_{B,dR}$ does not respect the rational structures.

   Example: $X = \mathbb{G}_m$ over $\mathbb{Q}$. $H^1_{dR}(\mathbb{G}_m, \mathbb{Q}) = \mathbb{Q} \cdot \frac{dz}{z}$, $H^{sing}_1(\mathbb{G}_m(\mathbb{C}), \mathbb{Q}) = \mathbb{Q} \cdot \gamma$ with γ the unit circle. Then $\int_\gamma \omega = 2\pi i \notin \mathbb{Q}$.

   "Periods" are the numbers appearing as coefficients of the matrix of $comp_{B,dR}$ with respect to rational bases for any choice of $X/\mathbb{Q}$.

3) Motivic interpretation.

   Goal: Construct a category of "motives" $MM_{Nori}(\mathbb{Q})$, which should enjoy the following properties:
   - ·) universal cohomology theory (compatible with singular cohomology);
   - ·) $MM_{Nori}(\mathbb{Q})$ is Tannakian (f.d. repr.'s of an algebraic group scheme);
   - ·) two fiber functors (Betti and de-Rham realization) $\omega_B, \omega_{dR}: MM_{Nori}(\mathbb{Q}) \to \mathbb{Q}\text{-Vec}$.

   There is a torsor of isomorphisms between these two functors, and a complex point of this yields the comparison isomorphism as above.

   The entries of these isomorphism are "periods".

Goal: Prove that these three definitions of periods coincide.

Roughly, the program for the seminar would consist of:

- An introduction to algebraic de Rham cohomology, mixed Hodge structures, ...
- construction of the category $\mathcal{MM}_{Nori}$ of Nori motives, with all its properties;
- prove the equivalence of the definition of periods.

Doesn't sound interesting? Well, I can imagine, but there would still be some advantages in learning these topics (other than writing down integrals in a fancier way...)
Here are some pros:

1) We would learn some Hodge theory, which doesn't harm anybody;
2) we would learn some motives. The ones of Nori are not just used for the purpose of studying periods, but they are a central theme in the theory of motives: conjecturally, $\mathcal{MM}_{Nori}$ should be the heart of a t-structure on Voevodsky's geometric motives (equivalently, $D(\mathcal{MM}_{Nori}) = DM_{gm}$).
If you are interested in becoming more familiar with the theory of motives, this seminar could serve as a gentle introduction
3) you would make Riccardo very happy.

The main reference for the seminar would be:
"Periods and Nori motives" by Annette Huber and Stefan Müller-Stach.