① p-adic Hodge theory

Motivation: for $X_{/F}$ smooth projective over a number field F, we have ℓ-adic $\mathrm{Gal}(\bar{F}/F)$ representations $H^n_{ét}(X_{\bar{F}}, \mathbb{Q}_\ell) =: V$ for ℓ a prime of $\mathbb{Q}$.
i.e. V is a finitely gen. $\mathbb{Q}_\ell$-vector space.

When we want to understand representations of a global Galois group, a good strategy is to localize and consider V as $\mathrm{Gal}(\bar{F}_\mathfrak{p}/F_\mathfrak{p})$-representation for every prime $\mathfrak{p}$ of F.

If $\mathfrak{p} \nmid \ell$ one can ~~understand~~ the above representation algebraically, but if $\mathfrak{p} | p$, then there is ~~one encounters~~ a huge amount of p-adic representations of p-adic fields and hence it is necessary a study of such representations in order to single out those with better properties (among which we have those "coming from geometry, i.e. of the form $H^n_{ét}(X_{\bar{F}_\mathfrak{p}}, \mathbb{Q}_p)$ as above)

The classification of p-adic representations of p-adic field can be addressed using the p-adic period rings (e.g. B_{dR}, B_{st}, ...) and the major part of the seminar would be on introducing those and see how they are related with (good) properties of p-adic representations of p-adic fields.

This is also strictly connected with comparison isomorphisms such as étale - De Rham $\quad B_{dR} \otimes_{\mathbb{Q}_p} H^r_{ét}(X_{\bar{\mathbb{Q}}_p}, \mathbb{Q}_p) \simeq B_{dR} \otimes_{\mathbb{Q}_p} H^r_{dR}(X)$.

A nice plan for the last part of the seminar might be to see the construction and main properties of the Fargues - Fontaine curve and to use it to prove the theorem "weakly admissible => admissible"

For the first part I plan to use: Brinon - Conrad: CMI Summer school notes on p-adic Hodge-theory [1]

For the F.F. curve there is an introductory exposition: De Shalit: The Fargues-Fontaine Curve and p-adic Hodge theory [2]
that explains Fargues and Fontaine's original paper (Courbes et fibrés vectoriels en théorie de Hodge p-adique)
Notice that the theorem weakly admissible => admissible is also contained in ([1], section 11), so, depending on the time and amount of material, we can use only [1], only [2] or compare.
I think that a geometric approach for the last part is probably more enjoyable for a seminar.