

② p-adic uniformization of Mumford curves

The seminar aim would be to explore the ~~follow~~ p-adic counterpart of the following situation.

Every compact curve X over \mathbb{C} of genus $g \geq 2$ is expressed as a quotient $X \simeq H_{\mathbb{C}} / \Gamma(X) =: \Gamma$ with $\Gamma \subset \mathrm{SL}_2(\mathbb{R})$ of a certain kind (e.g. can be presented with $2g$ generators and a single relation). Moreover every such curve is algebraic and its Jacobian $\mathrm{Jac}(X)$ is isomorphic to $\Gamma(X, \Omega_{X/\mathbb{C}}^1)^\vee / H_1(X, \mathbb{Z})$ which can also be shown to be an abelian variety. (C)

In the p-adic world we have the following picture:
for a smooth projective curve X over a finite extension K of \mathbb{Q}_p one can find a model over \mathcal{O}_K and look at the special fiber.
After possibly base changing with a finite extension of K (we replace K with such extension if needed) we have the following two possibilities:

- the special fiber is smooth
- the special fiber has only ordinary double points as singularities

In the second case (the one we are interested in in the seminar) there exists an open $\Omega \subset \mathbb{P}^1(K)$ and a subgroup $\Gamma \subset \mathrm{PGL}_2(K)$ st (the rigid analytification of) X is isomorphic to Ω / Γ (the quotient being taken also in the rigid analytic sense).

Moreover every quotient of this type is the rigid analytification of some proper curve over a p-adic field. ~~And~~ $\mathrm{Jac}(X)^{\mathrm{analytic}}$ is a rigid analytic abelian variety.

If we work over $\mathbb{C}_p = \widehat{\overline{\mathbb{Q}_p}}$ (that contains all finite extensions of \mathbb{Q}_p) the analogy with (C) is even more enlightening, as the prototypical open $\Omega \subset \mathbb{P}^1(\mathbb{C}_p)$ is $\Omega = \mathbb{P}^1(\mathbb{C}_p) - \mathbb{P}^1(\mathbb{Q}_p)$, the "p-adic upper half plane".

Plan of the seminar:

- Review of some rigid analytic geometry (in Tate's sense)
- Explore the setting described above

References: Geritzen, Van der Put : Schottky groups and Mumford curves
Lütkebohmert : Rigid geometry of curves and their Jacobians