**Algebraic Geometry I** Prof. Dr. Ulrich Görtz
WS 2025/26 Dr. Andreas Pieper

# Problem sheet 6

Due date: Dec. 2, 2025.

## Problem 19

Let $X$ be a topological space and let $\mathscr{F}$ be a sheaf on $X$. Let $U \subseteq X$ be an open subset and $s, t \in \mathscr{F}(U)$. Show that the set of $x \in U$ with $s_x = t_x \in \mathscr{F}_x$ is open in $X$.

## Problem 20

Let $X$ be a topological space and let $\varphi\colon \mathscr{F} \to \mathscr{G}$ be a morphism of sheaves on $X$. Show that the following are equivalent:

(i) for all open subsets $U \subseteq X$, the map $\varphi(U)\colon \mathscr{F}(U) \to \mathscr{G}(U)$ is bijective,

(ii) the morphism $\varphi$ is an isomorphism of sheaves,

(iii) for all $x \in X$, the map $\varphi\colon \mathscr{F}_x \to \mathscr{G}_x$ on the stalks is bijective.

**Problem 21** Let $k$ be a field. Consider $X = \mathbb{A}^2_k = \operatorname{Spec}(k[x, y])$ and $U = X \smallsetminus \{(0, 0)\}$. Compute $\mathcal{O}_X(U)$.

## Problem 22

Let $X$ be a topological space, and let $\mathscr{F}, \mathscr{G}$ be sheaves of abelian groups on $X$. Show that the presheaf $U \mapsto \mathscr{F}(U) \oplus \mathscr{G}(U)$ (with the obvious restriction maps) is a sheaf. (*Remark:* It is not hard to show that it satisfies the universal properties of product and of coproduct in the category of sheaves of abelian groups on $X$.)

(More difficult) bonus question: What about direct sums with infinite index set? What about direct product with infinite index set?